

Method 2: Factor as a Difference of Squares

This is a difference of squares, $a^2 - b^2$, with $a = (3x + 8)$ and $b = (x - 2)$.

$$\begin{aligned}(3x + 8)^2 - (x - 2)^2 &= [(3x + 8) + (x - 2)][(3x + 8) - (x - 2)] \\&= (3x + 8 + x - 2)(3x + 8 - x + 2) \\&= (4x + 6)(2x + 10) \\&= 2(2x + 3)[2(x + 5)] \\&= 4(2x + 3)(x + 5)\end{aligned}$$

Key Concepts

- Always look for a common factor first when factoring a trinomial.
- You can factor a difference of squares as $a^2 - b^2 = (a + b)(a - b)$.
- You can factor a perfect square trinomial as $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$.

Communicate Your Understanding

- C1** Use words and diagrams to explain why $x^2 + 9$ cannot be factored over the integers.
- C2** When her classmate showed Barbara the first step in Example 3b), $25k^2 - 60km + 36m^2 = (5k)^2 - 2(5k)(6m) + (6m)^2$, Barbara asked, "Where did the 2 come from?" Answer Barbara's question.

Practise

For help with questions 1 and 2, see Example 1.

1. Factor.

- | | |
|-----------------|------------------|
| a) $x^2 - 16$ | b) $y^2 - 100$ |
| c) $9k^2 - 36$ | d) $4a^2 - 121$ |
| e) $36w^2 - 49$ | f) $144p^2 - 1$ |
| g) $16n^2 - 25$ | h) $100g^2 - 81$ |

2. Factor.

- | | |
|--------------------|---------------------|
| a) $m^2 - 49n^2$ | b) $h^2 - 25d^2$ |
| c) $100 - 9c^2$ | d) $169a^2 - 49b^2$ |
| e) $25x^2 - 36y^2$ | f) $16c^2 - 9d^2$ |
| g) $162 - 8s^2$ | h) $75h^2 - 27g^2$ |

For help with question 3, see Example 2.

3. Verify that each trinomial is a perfect square. Then, factor.

- | | |
|----------------------|---------------------|
| a) $x^2 + 12x + 36$ | b) $k^2 + 18k + 81$ |
| c) $y^2 - 6y + 9$ | d) $m^2 - 14m + 49$ |
| e) $x^2 + 20x + 100$ | f) $64 - 16r + r^2$ |

For help with question 4, see Example 3.

4. Verify that each trinomial is a perfect square. Then, factor.

- | | |
|-------------------------|-----------------------|
| a) $4c^2 + 12c + 9$ | b) $16k^2 - 8k + 1$ |
| c) $25x^2 + 70x + 49$ | d) $9y^2 - 30y + 25$ |
| e) $100c^2 - 180c + 81$ | f) $25 + 80y + 64y^2$ |

Connect and Apply

5. Each of the following is not factorable over the integers. Why not?

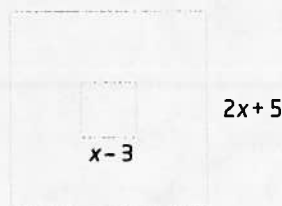
- a) $9x^2 - 16y$
- b) $36a^2 + 107a + 81$
- c) $10w^2 - 70wz + 49z^2$
- d) $25n^2 + 36m^2$

6. Factor fully, if possible.

- a) $4x^2 + 28xy + 49y^2$
- b) $9k^2 - 24km + 16m^2$
- c) $25p^2 + 60pq + 44q^2$
- d) $9y^2 - 7x^2$
- e) $2a^2 - 28ab + 98b^2$
- f) $196n^2 - 144m^2$
- g) $25x^2 + 70xy + 49y^2$
- h) $100f^2 - 120fg + 36g^2$
- i) $400p^3 - 900pq^2$

For help with question 7, see Example 4.

- 7. a) Find an algebraic expression for the area of the shaded region.
- b) Write the area expression in factored form.



8. Determine all values of b so that each trinomial is a perfect square.

- a) $y^2 + by + 121$
- b) $4x^2 + bx + 25$
- c) $9n^2 + bnp + 49p^2$
- d) $w^2 + 10w + b$
- e) $81m^2 - 90m + b$
- f) $16x^2 - 88xy + b^2y^2$

9. Determine two values of k so that each trinomial can be factored as a difference of squares.

- a) $m^2 - kn^2$
- b) $kx^2 - 9$
- c) $49c^2 - k$

10. Factor, if possible.

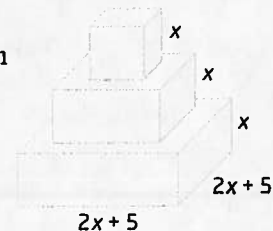
- a) $9a^2b^2 - 24abcd + 16c^2d^2$
- b) $225 - (x + 5)^2$
- c) $(3c + 2)^2 - (3c - 2)^2$
- d) $4x^2 + 26x + 9$

11. The area of an unknown shape is represented by $9x^2 + 30x + 25$. If x must be an integer, what shape(s) could this figure be?

12. A box is in the shape of a rectangular prism. Its volume is given as $x^3 - 2x^2 + x$.

- a) Determine algebraic expressions for the dimensions.
- b) Describe the faces of the box.

13. **Chapter Problem** In Section 5.3, question 12, you found an algebraic expression for the total of the top surface areas of the three prisms used to make the pedestal.



- a) Write algebraic expressions for the exposed top surface areas of the middle and bottom layers of the pedestal.
- b) Factor each expression from part a).
- c) Compare the expressions for the exposed surface areas when x represents 5 cm.

14. The radius of a circle has been decreased by a certain amount. Its area is now given as $\pi r^2 - 14\pi r + 49\pi$, where r was the original radius, in centimetres.

- a) What was the decrease in radius?
- b) What was the decrease in area?